STRENGTH OF MATERIALS

CIVIL ENGINEERING-CE

GATE / PSUs

STUDY MATERIAL

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ENGINEERS INSTITUTE OF INDIA

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CONTENT

1 CHAPTER

SIMPLE STRESSES AND STRAINS

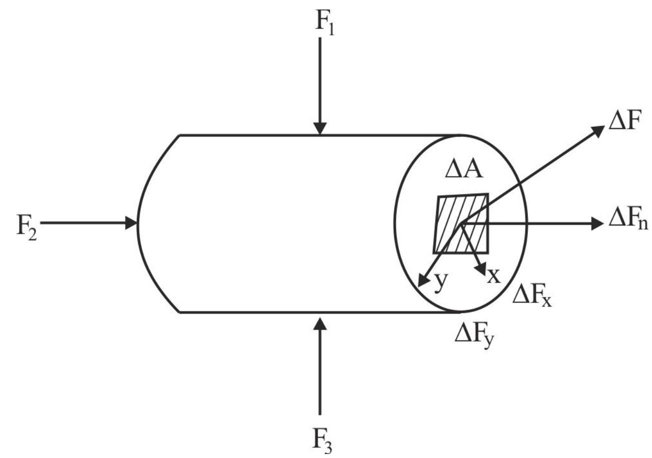
STRESS (σ):

It is the internal resistance offered by a body against the deformation numerically, it is given as force per unit area.

Stress on elementary area ΔA,

i.e. $$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} = \frac{dF}{dA} (N/m^2)$$ This unit is called

Pa(Pascal)



In case of normal stress dF always ⊥ (perpendicular) to area dA.

Pascal is a small unit in practice. These units are generally used

$1\,kPa = 10^3\,Pa = 10^3\,N/m^2$

$1\,MPa = 10^6\,Pa = 10^6\,N/m^2$

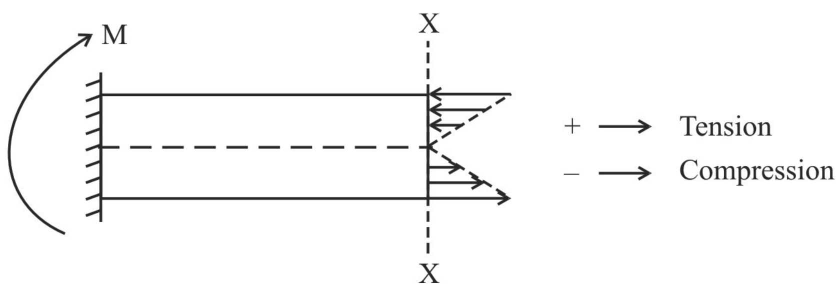
$1\,GPa = 10^9\,Pa = 10^9\,N/m^2$

1. **Normal Stress:** It may be tensile or compressive depending upon the force acting on the material.

   Tensile and compressive stresses are called **direct stresses.**

   When, $\sigma > 0$, Tensile
   When, $\sigma < 0$, Compressive

   ➢ The other types of normal stress is bending normal stress.



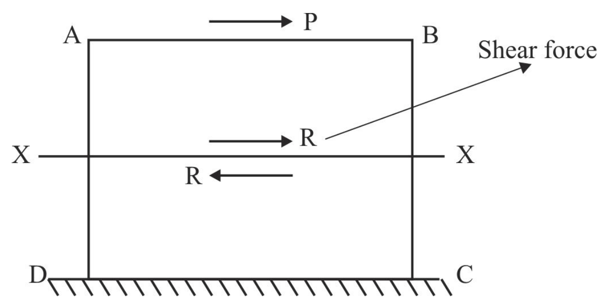
Bending stress are linearly distributed from zero at neutral axis to maximum at surface.

➢ In bending, the cross-sectional area rotates about transverse axis and the axis about which the cross-sectional area rotates is called neutral axis hence in bending, neutral axis is always transverse axis.

2. **Shear Stress (τ):** It is the intensity of shear resistance along a surface (Let X-X).

$$\boxed{\tau = \frac{Shear\ force}{Shear\ Area}}(N/m^2)$$

In case of shear stress force always parallel to the sheared area *i.e.* P is parallel to sheared area in figure.



3. **Conventional or Engineering Stress (σ_0):** It is defined as the ratio of load (P) to the original area of cross-section (A_0):

∴ $$\boxed{\sigma_0 = \frac{P}{A_0}}$$

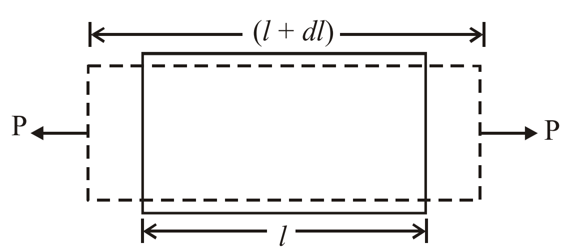
4. **True Stress (σ):** It is defined as the ratio of load (P) to the instantaneous area of cross-section (A):

∴ $\boxed{\sigma = \frac{P}{A}}$ $or,$ $\boxed{\sigma = \sigma_0(1+\varepsilon)}$ Where ε = strain $\left[Al = A_0 l_0\right]$ Initial volume = Final volume
$l = l_0(1+\varepsilon)$

STRAINS (ε):

It is defined as the change in length per unit length. It is a dimensionless quantity.

i.e. $$\boxed{\varepsilon = \frac{\text{change in length}}{\text{original length}} = \frac{dl}{l}}$$



1. **Conventional or Engineering strain:** It is defined as the change in length per unit original length.

$$\boxed{\varepsilon = \frac{l - l_0}{l_0}}$$

Where,

l = Deformed length $\quad\quad$ l_0 = Original length

e.g. from above figure.

$$\varepsilon = \frac{l + dl - l}{l} \qquad \boxed{\varepsilon = \frac{dl}{l}}$$

2. Natural Strain: It is defined as the change in length per unit instantaneous length.

$$\boxed{\bar{\varepsilon} = \int_{l_0}^{l} \frac{dl}{l} = \ln\frac{l}{l_0} = \ln(1+\varepsilon) = \ln\left(\frac{A_0}{A}\right) = 2\ln\left(\frac{d_0}{d}\right)}$$

Also, $\because$ $\bar{\varepsilon} = \ln(1+\varepsilon)$

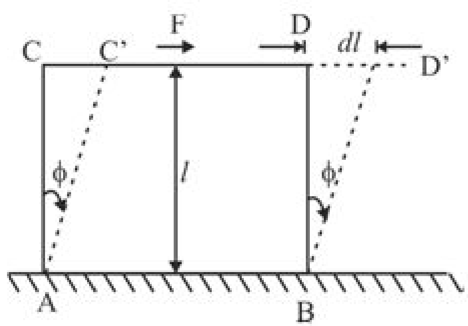
$\Rightarrow$ $1 + \varepsilon = e^{\bar{\varepsilon}}$

$\Rightarrow$ $\varepsilon = e^{\bar{\varepsilon}} - 1$

Volume of the specimen is a assumed to be constant during plastic deformation

$\because$ $\boxed{A_0L_0 = AL}$ -Valid till neck formation.

3. Shear Strain (ϕ): It is the strain produced under the action of shear stresses.



Shear Strain = tan ϕ

For small strain, $\boxed{\tan\phi \approx \phi}$

From figure, $\Delta ACC'$ or $\Delta BDD'$

$$\tan\phi = \frac{dl}{l} = \frac{CC'}{l}$$

$$\boxed{\phi = \frac{dl}{l} = \frac{\text{Transverse displacement}}{\text{Distance from lower face}}}$$

- Shear strain cause deformation in shape but volume remains same.

4. Superficial strain (ε_s): It is defined as the change in area of cross section per unit original area.

$$\boxed{\varepsilon_s = \frac{A - A_0}{A_0}}$$

Where, A = Final area A_0 = Original area

5. Volumetric Strain (ε_v): It is defined as the change in volume per unit original volume.

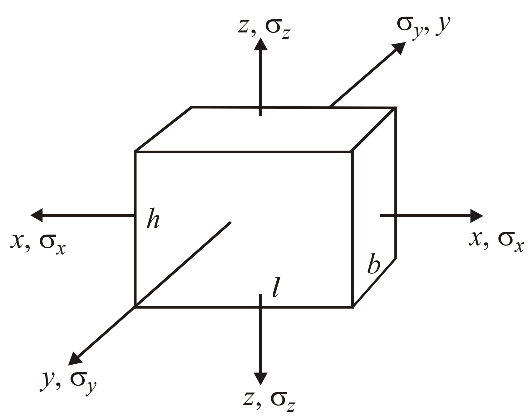
$$\varepsilon_V = \frac{V - V_0}{V_0}$$

Where, V = Final volume V_0 = Original volume

- Stress and strain are tensor (*neither vector nor scalar*) of 2nd order.

$$\text{Volumetric strain } \varepsilon_V = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

Volumetric strain for various shapes:



(*i*) Rectangular body:

V = lbh on partial differentiation

$$\delta V = \delta l(b.h) + \delta b(l.h) + \delta h(b.l)$$

$$\varepsilon_V = \frac{\delta V}{V} = \frac{\delta l}{l} + \frac{\delta b}{b} + \frac{\delta h}{h}$$

$$\varepsilon_V = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

Note: $\varepsilon_x, \varepsilon_y, \varepsilon_z$ are the strain corresponding to the stresses $\sigma_x, \sigma_y, \sigma_z$ in x-direction, y-direction, z-direction respectively

$$\varepsilon_V = \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1 - 2\upsilon)$$

$\mu \rightarrow$ Poisson Ratio

$\mu = 0.5$ For rubber

(*ii*) For cylindrical body:

$$V = \frac{\pi}{4} d^2 l$$

$$\delta V = 2\, dl \,.\, \delta d \,.\, \frac{\pi}{4} + \frac{\pi}{4} d^2 \delta l$$

$$\varepsilon_V = \frac{\delta V}{V} = 2\frac{\delta d}{d} + \frac{\delta l}{l}$$

$$\boxed{\varepsilon_V = 2\varepsilon_d + \varepsilon_l}$$

(*iii*) For spherical body

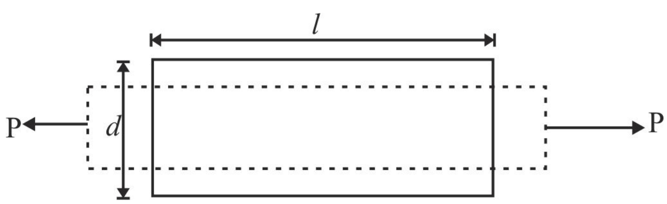
$$\boxed{\varepsilon_V = 3\frac{\delta d}{d}} \qquad \boxed{V = \frac{4}{3}\pi r^3} \qquad d = 2r$$

Gauge Length: It is that portion of the test specimen over which extension or deformation is measured. i.e. this length is used in calculating strain value.

Poisson's ratio $\left(\mu \text{ or } \frac{1}{m}\right)$: Value of μ varies between (–1 to 0.5)

The ratio of the lateral strain to longitudinal strain is called the Poisson's ratio.

$$\boxed{\mu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}} \quad \text{or} \quad \boxed{\mu = \frac{-\left(\frac{\delta d}{d}\right)}{\left(\frac{\delta l}{l}\right)}}$$



- For a given material, the value of 'μ' is constant throughout the linearly elastic range.
- For most of the metals the value of 'μ' lie between 0.25 – 0.42
- 'μ' varies from (–1 to 0.5)

Note: 'μ' for ductile material is greater than 'μ' for brittle metals.

Table

Material	Value of 'μ'	Remarks
Cork	0	∴ Used in bottle to withstand pressure
Foam	–1	
Rubber	0.5	
Concrete	0.1 – 0.2	
C.I.	0.23 – 0.27	

For cork $\mu = 0$

For rubber $\mu = 0.5$

For concrete $\mu = 0.1 - 0.2$

Isotropic Material: These materials have same elastic properties in all directions.

No. of independent elastic constants = 2, *i.e.* if any of 2 elastic constants is known then other can be derived.

Orthotropic Material: The number of independent elastic constants is 9.

Anisotropic materials: These materials don't have same elastic properties in all directions.

Elastic modulii will vary with additional stresses appearing. ∵ There is a coupling between shear stress and normal stress for an isotropic material.

The number of independent elastic constants is 21.

Hooke's Law: It states that when a material is loaded such that the intensity of stress is within a certain limit, the ratio of the intensity of stress to the corresponding strain is a constant which is characteristics of that material.

Note: The certain limit is called proportionality limit but for most practical purposes, proportionality limit equals elastic limit.

i.e. $\boxed{\dfrac{\text{Stress}}{\text{Strain}} = \text{Constant} = E}$ *i.e.,* $\sigma = E\varepsilon$

Where, E = Young's Modulus (N/m^2)

Or

Modulus of Elasticity

- For steel, value of E = 210 GPa (1 GPa = 10^3 N/m^2)
- For aluminum, value of E = 73 GPa $E_{Al} \simeq \dfrac{1}{3}\text{rd } E_{steel}$
- For Plastic, value of E = 1 GPa – 14 GPa

  Note : As flexibility increases, value of young's modulus decreases.

  It is resistance to elastic strain.

Shear Modulus of Elasticity OR Modulus of Rigidity (G or C): It is defined as the ratio of shearing stress to shearing strain.

$$\boxed{G \text{ or } C = \frac{\text{Shear stress}}{\text{Shear strain}}} \text{ i.e. } \tau = G\,\phi$$

Bulk Modulus (K):

It is defined as the ratio of uniform stress intensity to volumetric strain within the elastic limits.

$$\boxed{K = \frac{\text{Normal Stress}}{\text{Volumetric Strain}}}$$

Note: Elastic constant relationship

(i) $E = 2C(1+\mu)$, where, v = Poisson's ratio.

(ii) $E = 3K(1-2\mu)$

(iii) $\mu = \dfrac{3K-2C}{6K+2C}$

(iv) $E = \dfrac{9KC}{3K+C}$

STRESS-STRAIN DIAGRAM:

I. Ductile material (Mild Steel):

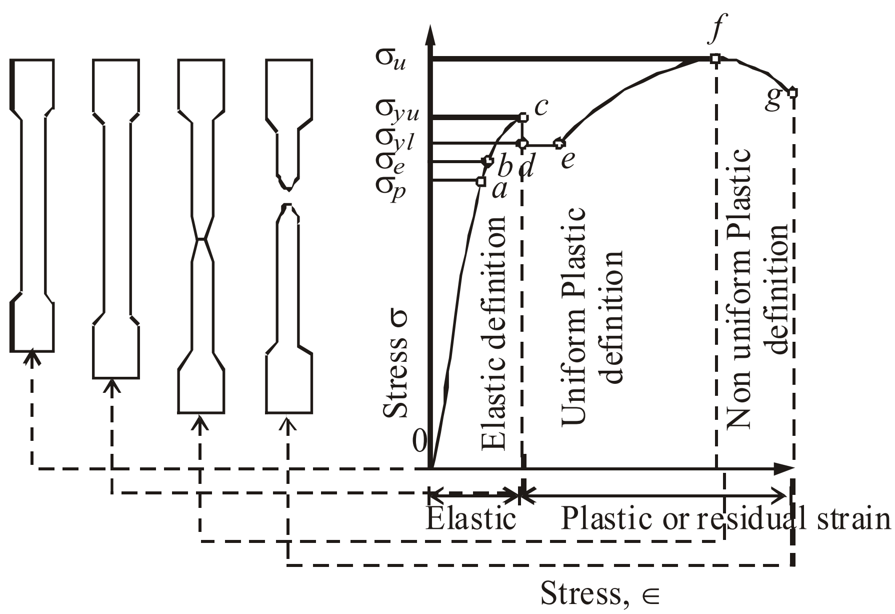


Figure: ***Typical stress-strain diagram for a ductile material***

- Point '*a*'→ Limit of proportionality: Up to this point '*a*', Hooke's law is obeyed; '*oa*' is a straight line.

  Stress corresponding to this point is called 'proportional limit stress, σ_p'

Comparison of Engineering and true stress strain curve:

- The true stress-strain curve is also known as **flow curve.**
- True stress-strain curve gives a true indication of deformation characteristics because it is **based on the instantaneous dimension of specimen.**
- In engineering stress-strain curve, the stress drops down after necking since it is **based on the original area.**
- In true stress strain curve, the stress however increases after necking since the cross section area of the specimen **decreases rapidly after necking.**
- The flow curve of many metals in the region of uniform plastic deformation can be expressed by **simple power law.**

$$\sigma_T = K(\in_T)^n$$

where, K is the strength co-efficient, σ_T is time stress.

n is the strain hardening coefficient.

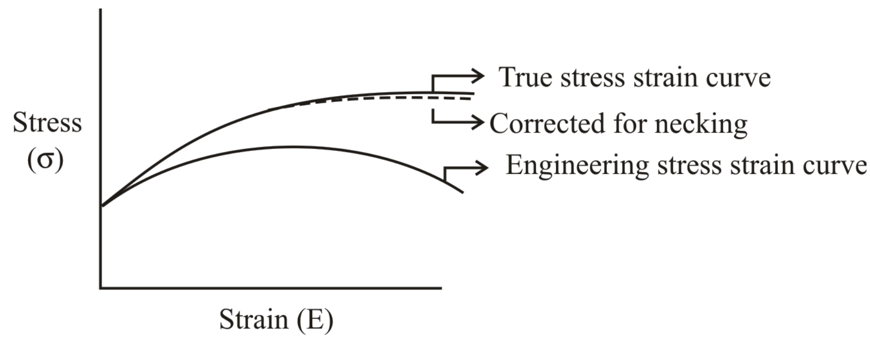
n = 0 for perfectly plastic solid

$n = 1$ In elastic solid

For most metals $0.1 < n < 0.5$

$\sigma_{True} > \sigma_{Nominal} \rightarrow$ if force is tensile, since area decreases.

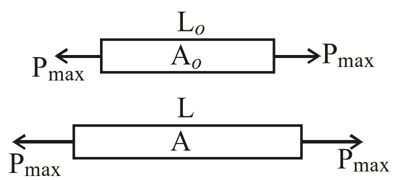
$\sigma_{True} > \sigma_{Nominal} \rightarrow$ if force is compressive, since area increase.



- Relation between ultimate tensile strength and true stress at maximum load.

Ultimate tensile strength $\sigma_u = \frac{P_{max}}{A_o}$

True stress at maximum load = $(\sigma_u)_T = \frac{P_{max}}{A}$



True strain at max load $(\varepsilon_T) = \ln\frac{A_o}{A}$ or $\frac{A_o}{A} = e^{\epsilon_T}$

Eliminating P_{max} we get

$$(\sigma_u)_T = \frac{P_{max}}{A} \times \frac{A_o}{A_o}$$

$$= \frac{P_{max}}{A_o} \times e^{\epsilon_T}$$

$$\boxed{\Rightarrow (\sigma_u)_T = \sigma_u\, e^{\epsilon_T}}$$

Here, P_{max} is the max force.

A_o = original cross section area

A = instantaneous cross section area

- Based on the above theory two examples has been provided.